On the dispersionless Kadomtsev-Petviashvili equation in n+1 dimensions: exact solutions, the Cauchy problem for small initial data and wave breaking

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Abstract:

We study the $n+1$-dimensional generalization of the dispersionless Kadomtsev-Petviashvili (dKP) equation, a universal equation describing the propagation of weakly nonlinear, quasi one dimensional waves in $n+1$ dimensions, and arising in several physical contexts, like acoustics, plasma physics and hydrodynamics. For $n=2$, this equation is integrable, and it has been recently shown to be a prototype model equation in the description of the two dimensional wave breaking of localized initial data. We construct an exact solution of the $n+1$ dimensional model containing an arbitrary function of one variable, corresponding to its parabolic invariance, describing waves, constant on their paraboloidal wave front, breaking simultaneously in all points of it. Then we use such solution to build a uniform approximation of the solution of the Cauchy problem, for small and localized initial data, showing that such a small and localized initial data evolving according to the $n+1$-dimensional dKP equation break, in the long time regime, if and only if $1 \leq n \leq 3$; i.e., in physical space. Such a wave breaking takes place, generically, in a point of the paraboloidal wave front, and the analytic aspects of it are given explicitly in terms of the small initial data.

References:

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